NWP Equations (continued)

(Adapted from UCAR/COMET Online Modules)

Prognostic/Diagnostic Equations

Equations (1a), (1b), (3), and (4) are called prognostic equations because time changes in forecast variables (u, v, T, and q) are determined explicitly using dynamic forcing equations. In equations (2) and (5), the remaining variables (ω and z) are determined from the prognostic variables. Because they do not calculate time changes directly, they are known as diagnostic equations.

Physical Processes

All of the forecast equations must try to account for the effects of processes that cannot be forecast directly by the models, due to the complexity of the physical processes being simulated (for example, radiation) or because the actual processes occur at scales too small to be included directly in the model (for example, convective clouds). Shorthand notations for the empirical approximations used in the model appear as F_x, F_y, H, E, and P in the forecast equations.

F_x and F_y (in equations 1a and 1b) are "friction" terms that modify the wind via surface drag but also incorporate other processes, including horizontal and vertical transport of momentum by turbulent eddies (generally called diffusion in large-scale models). "Friction" is affected by vegetation type (trees versus grass), surface type (snow and water), surface temperature, and other conditions.

The diabatic heating term H (in equation 3) incorporates several processes:

$$H = H_L + H_C + H_R + H_S$$

where

- H_L is latent heating caused by condensation in large-scale ascent of saturated, stably-stratified air or cooling due to evaporation of falling precipitation and evaporation of water at the surface
- H_C is latent heating due to condensation occurring in convection (which may itself be approximated)
- H_R is the radiative heating rate (primarily at the surface for solar radiation and within moist layers of the atmosphere for infrared radiation; H_R is negative for radiative cooling)

- H_S represents sensible heat flux to and from the surface of the earth

The precipitation rate, $P = P_L + P_C$ (stratiform and convective precipitation), is closely linked to H_L and H_C. Their calculation depends on details such as whether the model predicts clouds and which convective parameterization or microphysics parameterization is used.

Evaporation (E) can be due either to evaporative moisture flux from the earth's surface or the evaporation of precipitation before it reaches the ground.

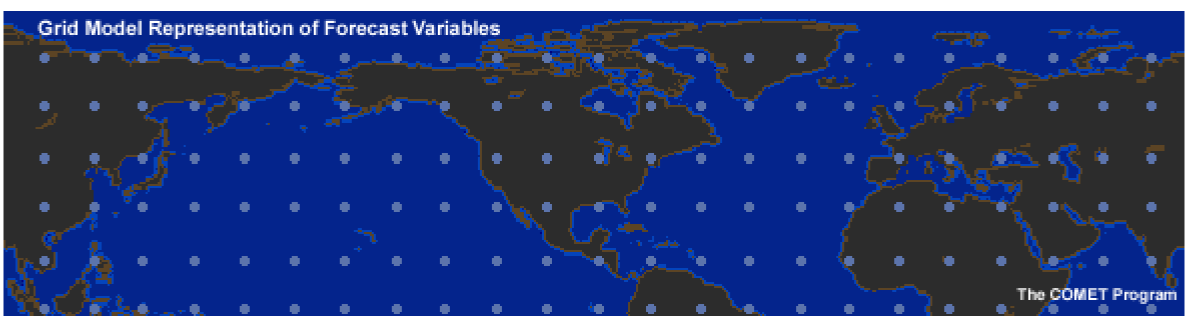
To this point, the discussions have been based on flow over a flat surface. The effects of mountains must also be included in a model. They are accounted for in the choice of a vertical coordinate (discussed in the Vertical Coordinate and Vertical Resolution sections).

The degree to which NWP models can simulate the real atmosphere using these approximations has a direct impact on the amount of error in the model forecast in areas where these processes are occurring.

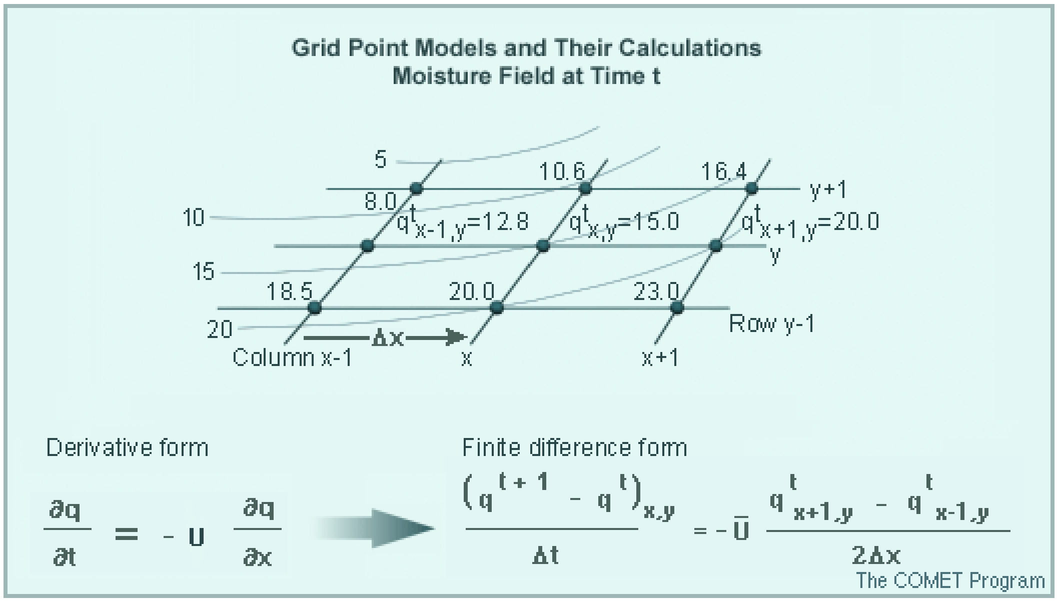
How Models Solve the Forecast Equations

Numerical models solve the forecast equations using one of two basic model formulations: grid point or spectral.

Grid point models solve the forecast equations at regularly spaced grid points. The forecast variables are specified on a set of grid points (illustrated below).



Derivatives are approximated at each grid point using a variety of arithmetic techniques called finite differences, as illustrated below. The choice of finite difference method affects both the computational error and amount of computer time required to run the model.

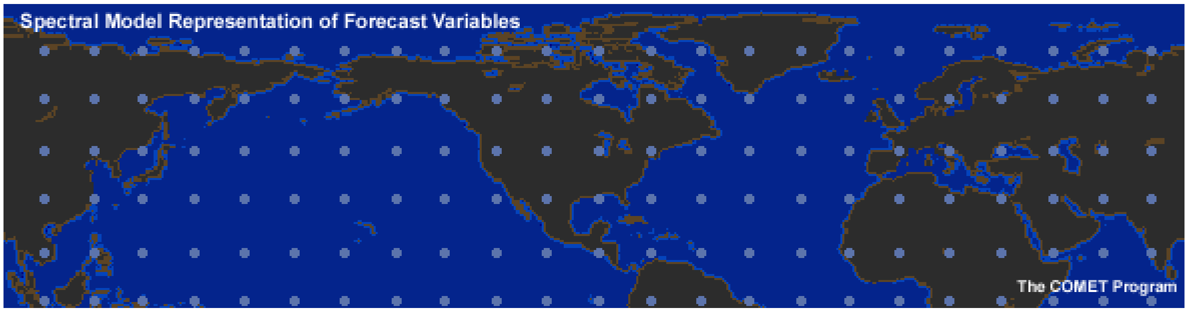


Spectral models are also based on the primitive equations, but their mathematical formulation and numerical solutions are quite different from grid point models for some of the forecast variables. Spectral techniques were developed as a means of increasing the speed and therefore enhancing the resolution of global models. Although these techniques can be adapted to limited-area (regional) forecast problems, they are most suited to global forecasting. However, as the resolution of global models increases over the next decade, the advantages of using spectral techniques may lessen and more global models may begin using grid point formulations.

Representing the Forecast Variables

Conceptually, spectral models emulate the process of drawing contours through a data field to represent the forecast variables. Instead of using grid points, they use a combination of continuous waves of differing wavelength and amplitude to specify the forecast variables and their derivatives at all locations (not just at grid points).

Consider, for example, the process of drawing contours through the same data shown in the grid point graphic for use in a two-wave model.



A typical contouring process would begin by defining the largest scale variations in the field by drawing a smooth "first guess" contour, as shown by the red curve.

In effect, the model has drawn a wave describing the longest wavelength feature in this data set. The next step will add smaller-scale refinements to this large-scale pattern, as shown by the green curve.

Now, the model has added detail by including two sets of local increases and decreases (+ and - areas, respectively) to the "first-guess" contour. In effect, the model has defined a second set of waves with a shorter wavelength and smaller amplitude (two sets of more subtle variations stretching around the globe, indicated by the positive and negative departures from the first wave), which complete the mathematical description of this data set for this two-wave model.

Since spectral models represent some of the forecast variables with continuous waves (a combination of sines and cosines) rather than at separate points along a wave, they can use more accurate numerical techniques to solve some of the equations and much longer forecast time steps than the finite difference techniques used by grid point models. The larger time step compensates for the added complexity of the computations required to solve the trigonometric functions. Since some grid point calculations are required in spectral models, some computational errors associated with grid point models will still be present.